

Sound Mutable and Const Types

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Abstract

Subsequent to the work presented in Sound and Complete Type Inference in BitC, we introduced by-reference parameters and discovered that the definition of mutability described in that report was insufficient. The language specification was updated to have a path-wise notion of mutability. At the same time, a `const` meta-constructor was introduced into the language to strip shallow mutability (up to the ref boundary) from existing types.

The following type rules reflect the modifications to the language associated with that change.

Language grammar:

Identifiers	$x ::= y \mid z \mid \dots$
Booleans	$b ::= true \mid false$
Indices	$i ::= 1 \mid 2 \mid !i$
Opt. Const	$e ::= e \mid [e]$
Values	$v ::= () \mid b \mid \lambda x.e \mid (v, v) \mid \ell$
Syn. Value	$v ::= v \mid x \mid 1 \mid (v, v)$
Left Expr	$l ::= x \mid 1 \mid e^\wedge \mid l.i$
Expressions	$e ::= v \mid e \mid e \mid l := e \mid \text{if } e \text{ then } e \text{ else } e$ $\mid \text{dup}(e) \mid e^\wedge \mid (e, e) \mid e.i$ $\mid \text{let}^\varkappa x = e \text{ in } e$
Let-kinds	$\varkappa ::= - \mid \kappa \mid \psi \mid \forall$
Locations	$L ::= 1 \mid \ell$

Types grammar:

Type Variables	$\alpha ::= \alpha \mid \beta \mid \gamma \mid \delta \mid \varepsilon \mid \dots$
M-Vars	$\varsigma ::= \alpha \mid \Psi\alpha$
Types.1	$\rho ::= \alpha \mid \text{unit} \mid \text{bool} \mid \tau \rightarrow \tau \mid \tau \times \tau$
Ref/Pointer	$\mid \uparrow\tau$
Mutable, const	$\mid \Psi\rho \mid [\tau]$
Types.2	$\varrho ::= \rho \mid \alpha \downarrow \rho$
Types	$\tau ::= \varrho \mid \varsigma \downarrow \rho$
Type Scheme	$\sigma ::= \tau \mid \forall \bar{\alpha}. \tau \setminus \mathcal{D}$
Poly. Constraints	$d ::= \star_x^\varkappa(\tau)$
Poly. Constraint Sets	$\mathcal{D} ::= \emptyset \mid \{\bar{d}\} \mid \mathcal{D} \cup \mathcal{D}$

Grammar for Dynamic Semantics:

Stack	$S ::= \emptyset \mid S, l \mapsto v$
Heap	$H ::= \emptyset \mid H, \ell \mapsto v$
Selection Path	$p ::= i \mid p.p$
Ivalues	$\mathcal{L} ::= 1 \mid \ell^\wedge \mid 1.p \mid \ell^\wedge.p$
Redex	$\mathcal{R} ::= - \mid \mathcal{R} e \mid v \mathcal{R} \mid \mathcal{L} := \mathcal{R}$ $\mid \text{if } \mathcal{R} \text{ then } e \text{ else } e \mid \mathcal{R}^\wedge$ $\mid \text{dup}(\mathcal{R}) \mid \text{dup}([\mathcal{R}]) \mid (\mathcal{R}, e)$ $\mid (v, \mathcal{R}) \mid \mathcal{R}.i \mid \text{let}^\varkappa \underline{x} = \mathcal{R} \text{ in } e$

Grammar for Static Semantics:

Unf. Constraints	$u ::= \tau = \tau \mid \kappa = \varkappa \mid d$
Unf. Constraint Sets	$\mathcal{C} ::= \emptyset \mid \{\bar{u}\} \mid \mathcal{C} \cup \mathcal{C} \mid \mathcal{C} \cup \mathcal{C}$
Substitutions	$\theta ::= \langle \rangle \mid [\alpha \mapsto \tau] \mid [\kappa \mapsto \varkappa] \mid \theta \circ \theta$
Binding Environment	$\Gamma ::= \emptyset \mid \Gamma, x \mapsto \sigma$
Store Typing	$\Sigma ::= \emptyset \mid \Sigma, L \mapsto \tau$

Definition 1 (Mutability Normalization in Composite Types).

$\mathbb{M}(\tau) \mid \text{Immut}(\tau)$	$= \perp$
$\mathbb{M}(\alpha)$	$= \perp$
$\mathbb{M}(\Psi\alpha)$	$= \perp$
$\mathbb{M}(\Psi\text{bool})$	$= \langle \rangle$
$\mathbb{M}(\Psi\text{unit})$	$= \langle \rangle$
$\mathbb{M}(\Psi(\tau_1 \rightarrow \tau_2))$	$= \langle \rangle$
$\mathbb{M}(\Psi\uparrow\tau)$	$= \langle \rangle$
$\mathbb{M}(\Psi(\tau_1 \times \tau_2))$	$= \mathbb{M}(\tau_1) \circ \mathbb{M}(\tau_2)$
$\mathbb{M}(\alpha \downarrow \rho)$	$= [\alpha \mapsto \Psi\rho] \circ \mathbb{M}(\Psi\rho)$
$\mathbb{M}(\Psi\alpha \downarrow \rho)$	$= \langle \rangle$
$\mathbb{M}(\alpha \downarrow \rho)$	$= [\alpha \mapsto \Psi\beta] \mid \frac{\text{new}}{\text{new}} \beta$

Definition 2 (Normalization of Const Types).

$\mathbb{N}(\alpha)$	$= \alpha$
$\mathbb{N}([\alpha])$	$= [\alpha]$
$\mathbb{N}(\text{bool})$	$= \text{bool}$
$\mathbb{N}([\text{bool}])$	$= \text{bool}$
$\mathbb{N}(\text{unit})$	$= \text{unit}$
$\mathbb{N}([\text{unit}])$	$= \text{unit}$
$\mathbb{N}(\tau_1 \rightarrow \tau_2)$	$= \mathbb{N}(\tau_1) \rightarrow \mathbb{N}(\tau_2)$
$\mathbb{N}([\tau_1 \rightarrow \tau_2])$	$= \mathbb{N}(\tau_1) \rightarrow \mathbb{N}(\tau_2)$
$\mathbb{N}(\uparrow\tau)$	$= \uparrow\mathbb{N}(\tau)$
$\mathbb{N}([\uparrow\tau])$	$= \uparrow\mathbb{N}(\tau)$
$\mathbb{N}(\Psi\tau)$	$= \Psi\mathbb{N}(\tau)$
$\mathbb{N}([\Psi\tau])$	$= \mathbb{N}([\tau])$
$\mathbb{N}(\tau_1 \times \tau_2)$	$= \mathbb{N}(\tau_1) \times \mathbb{N}(\tau_2)$
$\mathbb{N}([\tau_1 \times \tau_2])$	$= [\mathbb{N}(\tau_1) \times \mathbb{N}(\tau_2)]$
$\mathbb{N}(\alpha \downarrow \rho)$	$= \alpha \downarrow \mathbb{N}(\Psi\rho)$
$\mathbb{N}([\alpha \downarrow \rho])$	$= [\alpha \downarrow \mathbb{N}(\Psi\rho)]$
$\mathbb{N}(\varsigma \downarrow \rho)$	$= \varsigma \downarrow \mathbb{N}(\rho)$
$\mathbb{N}([\varsigma \downarrow \rho])$	$= [\varsigma \downarrow \mathbb{N}(\rho)]$

Definition 3 (Notational convenience). We write $\tau_1 \overset{\nabla}{=} \tau_2$ as shorthand for $\nabla(\tau_1) = \nabla(\tau_2)$, $\tau_1 \overset{\forall}{=} \tau_2$ for $\forall(\tau_1) = \forall(\tau_2)$, and $\tau_1 \overset{\circ}{=} \tau_2$ for $\circ(\tau_1) = \circ(\tau_2)$

Definition 4 (Const-ness Requirement).

$\mathcal{P}(e, \tau)$	$= \text{true}$
$\mathcal{P}([e], [\tau])$	$= \text{true}$
$\mathcal{P}(\underline{e}, [\tau])$ (otherwise)	$= \text{false}$
$\mathbb{P}(e, \tau)$	$= \tau$
$\mathbb{P}([e], \tau)$	$= [\tau]$

Rule	Pre-conditions	Evaluation Step
E-Rval	$S(l) = v$	$S; H; l \Rightarrow S; H; v$
E-#	$S; H; e \Rightarrow S'; H'; e'$	$S; H; \mathcal{R}[e] \Rightarrow S'; H'; \mathcal{R}[e']$
E-App	$l \notin \text{dom}(S)$	$S; H; \lambda \underline{x}. e v \Rightarrow S; l \mapsto v; H; e[l/x]$
E-If	$b_1 = \text{true} \quad b_2 = \text{false}$	$S; H; \text{if } \bar{b}_i \text{ then } e_1 \text{ else } e_2 \Rightarrow S; H; e_i$
E-i		$S; H; (v_1, v_2).i \Rightarrow S; H; v_i$
E-Dup	$l \notin \text{dom}(H)$	$S; H; \text{dup}(\underline{v}) \Rightarrow S; H, l \mapsto v; l$
EL- \wedge #	$S; H; e \Rightarrow S'; H'; e'$	$S; H; e^\wedge \Rightarrow S'; H'; e'^\wedge$
E- \wedge	$H(l) = v$	$S; H; l^\wedge \Rightarrow S; H; v$
E-:=#	$S; H; l \Rightarrow S'; H'; l'$	$S; H; l := e \Rightarrow S'; H'; l' := e$
E-:=Stack		$S, l \mapsto v; H; l := v' \Rightarrow S, l \mapsto v'; H; ()$
E-:=Heap		$S; H, l \mapsto v; l^\wedge := v' \Rightarrow S; H, l \mapsto v'; ()$
E-:=S.p	$v'_{i_i} = v_{i_i} \quad S, l \mapsto v_i; H; l.p := v'_i$ $\Rightarrow S, l \mapsto v'_i; H; ()$	$S, l \mapsto (v_1, v_2); H; l.i.p := v'_i$ $\Rightarrow S, l \mapsto (v'_1, v'_2); H; ()$
E-:=H.p	$v'_{i_i} = v_{i_i} \quad S; H, l \mapsto v_i; l^\wedge.p := v'_i$ $\Rightarrow S; H, l \mapsto v'_i; ()$	$S; H, l \mapsto (v_1, v_2); l^\wedge.i.p := v'_i$ $\Rightarrow S; H, l \mapsto (v'_1, v'_2); ()$
E-Let-M	$l \notin \text{dom}(S)$	$S; H; \text{let}^\psi \underline{x} = v_1 \text{ in } e_2 \Rightarrow S, l \mapsto v_1; H; e_2[l/x]$
E-Let-P		$S; H; \text{let}^\forall \underline{x} = v_1 \text{ in } e_2 \Rightarrow S; H; e_2[v_1/x]$

Figure 1. Small Step Operational Semantics

τ	$\Delta(\tau)$	$\nabla(\tau)$	$\blacktriangle(\tau)$	$\blacktriangledown(\tau)$	$\diamond(\tau)$	$\mathcal{J}(\tau)$	$\{\tau\}$
α	$\Psi\alpha$	α	$\Psi\alpha$	α	α	α	$\{\alpha\}$
unit	Ψunit	unit	Ψunit	unit	unit	unit	\emptyset
bool	Ψbool	bool	Ψbool	bool	bool	bool	\emptyset
$\tau_1 \rightarrow \tau_2$	$\Psi(\tau_1 \rightarrow \tau_2)$	$\tau_1 \rightarrow \tau_2$	$\Psi(\tau_1 \rightarrow \tau_2)$	$\tau_1 \rightarrow \tau_2$	$\tau_1 \rightarrow \tau_2$	$\tau_1 \rightarrow \tau_2$	$\{\tau_1\} \cup \{\tau_2\}$
$\uparrow\tau$	$\Psi\uparrow\tau$	$\uparrow\tau$	$\Psi\uparrow\tau$	$\uparrow\tau$	$\uparrow\tau$	$\uparrow\mathcal{J}(\tau)$	$\{\tau\}$
$\Psi\rho$	$\Delta(\rho)$	$\nabla(\rho)$	$\blacktriangle(\rho)$	$\blacktriangledown(\rho)$	$\diamond(\rho)$	$\mathcal{J}(\rho)$	$\{\rho\}$
$\tau_1 \times \tau_2$	$\Psi(\Delta(\tau_1) \times \Delta(\tau_2))$	$\nabla(\tau_1) \times \nabla(\tau_2)$	$\Psi(\tau_1 \times \tau_2)$	$\tau_1 \times \tau_2$	$\diamond(\tau_1) \times \diamond(\tau_2)$	$\mathcal{J}(\tau_1) \times \mathcal{J}(\tau_2)$	$\{\tau_1\} \cup \{\tau_2\}$
$\alpha \downarrow \rho$	$\Delta(\rho)$	$\nabla(\rho)$	$\blacktriangle(\rho)$	$\blacktriangledown(\rho)$	$\diamond(\rho)$	$\mathcal{J}(\rho)$	$\{\alpha \downarrow \rho\} \cup \{\rho\}$
$\varsigma \downarrow \rho$	$\Delta(\rho)$	$\nabla(\rho)$	$\blacktriangle(\varsigma) \downarrow \rho$	$\blacktriangledown(\varsigma) \downarrow \rho$	$\diamond(\rho)$	$\mathcal{J}(\rho)$	$\{\varsigma \downarrow \rho\} \cup \{\rho\}$
$\lfloor \tau \rfloor$	$\Delta(\tau)$	$\lfloor \tau \rfloor$	$\Psi \lfloor \tau \rfloor$	$\lfloor \tau \rfloor$	$\diamond(\tau)$	$\mathcal{J}(\rho)$	$\{\tau\}$

τ	Mut(τ)	Immut(τ)	Const(τ)	$\theta(\tau)$
α	false	false	false	τ if $[\alpha \mapsto \tau] \in \theta$, else α .
unit	false	true	true	unit
bool	false	true	true	bool
$\tau_1 \rightarrow \tau_2$	false	true	true	$\theta(\tau_1) \rightarrow \theta(\tau_2)$
$\uparrow\tau$	Mut(τ)	Immut(τ)	true	$\uparrow\theta(\tau)$
$\Psi\rho$	true	false	false	$\Psi\theta(\rho)$
$\tau_1 \times \tau_2$	Mut(τ_1) \vee Mut(τ_2)	Immut(τ_1) \wedge Immut(τ_2)	Const(τ_1) \wedge Const(τ_2)	$\theta(\tau_1) \times \theta(\tau_2)$
$\alpha \downarrow \rho$	Mut($\blacktriangledown(\rho)$)	false	false	$\alpha' \downarrow \theta(\rho)$ if $\theta(\alpha) = \alpha'$ ρ' if $\theta(\alpha) = \rho' \neq \alpha'$
$\varsigma \downarrow \rho$	Mut(ς) \vee Mut($\nabla(\rho)$)	false	false	$\varsigma' \downarrow \theta(\rho)$ if $\theta(\varsigma) = \varsigma'$ ϱ if $\theta(\varsigma) = \varrho \neq \varsigma'$
$\lfloor \tau \rfloor$	false	Immut($\nabla(\tau)$)	true	$\lfloor \theta(\tau) \rfloor$

τ	$\square(\tau)$	$\square(\tau)$	$\boxplus(\tau)$	$\mathbf{N}(\tau)$	$\mathfrak{M}(\tau)$
α	false	false	false	α	\diamond
unit	true	true	true	unit	\diamond
bool	true	true	true	bool	\diamond
$\tau_1 \rightarrow \tau_2$	true	true	true	$\mathbf{N}(\tau_1) \rightarrow \mathbf{N}(\tau_2)$	$\mathfrak{M}(\tau_1) \circ \mathfrak{M}(\tau_2)$
$\uparrow\tau$	$\square(\tau)$	true	true	$\uparrow\mathbf{N}(\tau)$	$\mathfrak{M}(\tau)$
$\Psi\rho$	$\square(\rho)$	$\square(\rho)$	$\boxplus(\rho)$	$\Psi\mathbf{N}(\rho)$	$\mathfrak{M}(\rho)$
$\tau_1 \times \tau_2$	$\square(\tau_1) \wedge \square(\tau_2)$	$\square(\tau_1) \wedge \square(\tau_2)$	$\boxplus(\tau_1) \wedge \boxplus(\tau_2)$	$\mathbf{N}(\tau_1) \times \mathbf{N}(\tau_2)$	$\mathfrak{M}(\tau_1) \circ \mathfrak{M}(\tau_2)$
$\alpha \downarrow \rho$	$\square(\rho)$	$\square(\rho)$	false	$\alpha \downarrow \mathbf{N}(\rho)$	$\mathfrak{M}(\rho)$
$\varsigma \downarrow \rho$	$\square(\rho)$	$\square(\rho)$	false	$\varsigma \downarrow \mathbf{N}(\rho)$	$\mathfrak{M}(\rho) \circ [\alpha \mapsto \Delta(\rho)]$ if $\varsigma = \Psi\alpha$ and $\square(\rho)$; $\mathfrak{M}(\rho)$ otherwise
$\lfloor \tau \rfloor$	$\square(\tau)$	$\square(\tau)$	true	$\nabla(\tau)$ if $\boxplus(\tau)$ $\lfloor \tau \rfloor$ otherwise	$\mathfrak{M}(\tau)$

Meanings of Operators

Δ	Add mutability up to the ref/function boundary.
∇	Remove mutability up to the ref/function boundary.
\blacktriangle	Add top-most mutability.
\blacktriangledown	Remove top-most mutability.
\diamond	Remove mutability and const up to the ref/function boundary.
\mathcal{J}	Remove mutability deeply up to the function boundary.
$\{\} \}$	Constraint Collection
Mut	Is the type <i>observably</i> mutable?
Immut	Is the type <i>effectively</i> deeply immutable?
Const	Is the type <i>effectively</i> const?
$\theta(\langle \rangle)$	Substitution.
\square	Is the type <i>concretizable</i> ? (i.e. can be made invariable by a substitution <i>only</i> for mutability-variables?)
\square	Is the type <i>concretizable</i> up to a reference boundary?
\boxplus	Is this type concrete enough to drop an enclosing const?
\mathbf{N}	Partially normalized form of const types, when a const wrapper can be dropped (ex: $\lfloor \text{bool} \rfloor \equiv \text{bool}$)
\mathfrak{M}	Produce a substitution to normalize $\varsigma \downarrow \rho$ types. (ex: $\Psi\alpha \downarrow \text{bool} \equiv \Psi\text{bool}$)
\mathfrak{M}	Produce a substitution to propagate mutability inwards into composite types (or fail with an error).
\mathbf{N}	Normalized form of const types, where const only appears around variable or constrained types.

Figure 2. Operations and Predicates on Types

S-Refl $\frac{}{\tau \trianglelefteq \tau}$	S-Trans $\frac{\tau_0 \trianglelefteq \tau_1 \quad \tau_1 \trianglelefteq \tau_2}{\tau_0 \trianglelefteq \tau_2}$	S-Mut $\frac{\rho \trianglelefteq \rho'}{\Psi \rho \trianglelefteq \Psi \rho'}$	S-Pair $\frac{\tau_1 \trianglelefteq \tau'_1 \quad \tau_2 \trianglelefteq \tau'_2}{\tau_1 \times \tau_2 \trianglelefteq \tau'_1 \times \tau'_2}$	S-MT1 $\frac{\nabla(\rho) \trianglelefteq \tau}{\alpha \downarrow \rho \trianglelefteq \tau}$	S-MT2 $\frac{\tau \trianglelefteq \blacktriangle(\rho)}{\tau \trianglelefteq \alpha \downarrow \rho}$
S-MF1 $\frac{\nabla(\rho) \trianglelefteq \tau}{\alpha \downarrow \rho \trianglelefteq \tau}$	S-MF2 $\frac{\alpha \downarrow \rho \trianglelefteq \rho'}{\Psi \alpha \downarrow \rho \trianglelefteq \Psi \rho'}$	S-MF3 $\frac{\tau \trianglelefteq \Delta(\rho)}{\tau \trianglelefteq \varsigma \downarrow \rho}$	S-Const1 $\frac{\tau \trianglelefteq \nabla(\tau')}{\tau \trianglelefteq [\tau']}$	S-Const2 $\frac{\nabla(\tau) \trianglelefteq \tau'}{[\tau] \trianglelefteq \tau'}$	

Figure 3. Copy Coercion Rules

T-Unit $\frac{}{\emptyset; \Gamma; \Sigma \vdash () : \text{unit}}$	T-Bool $\frac{}{\emptyset; \Gamma; \Sigma \vdash \text{b} : \text{bool}}$	T-Id $\frac{\Gamma(x) = \nabla \bar{\alpha}. \tau \setminus \mathcal{D} \quad \theta \Vdash \{\tau, \mathcal{D}\} \quad \text{dom}(\theta) = \{\bar{\alpha}\}}{\theta(\mathcal{D}); \Gamma; \Sigma \vdash x : \theta(\tau)}$
T-Hloc $\frac{\Sigma(\ell) = \tau}{\emptyset; \Gamma; \Sigma \vdash \ell : \uparrow \tau}$	T-Sloc $\frac{\Sigma(l) = \tau}{\emptyset; \Gamma; \Sigma \vdash l : \tau}$	T-Lambda $\frac{\mathcal{D}; \Gamma, x \mapsto \tau_1; \Sigma \vdash e : \tau_2 \quad \tau_1 \overset{\circ}{=} \tau'_1 \quad \tau_2 \overset{\circ}{=} \tau'_2 \quad \mathcal{P}(x, \tau_1)}{\mathcal{D}; \Gamma; \Sigma \vdash \lambda \underline{x}. e : \tau'_1 \rightarrow \tau'_2}$
T-App $\frac{\mathcal{D}_1; \Gamma; \Sigma \vdash e_1 \trianglelefteq \tau_a \rightarrow \tau_r \quad \mathcal{D}_2; \Gamma; \Sigma \vdash e_2 \trianglelefteq \nabla(\tau_a) \quad \Delta(\tau_r) \trianglelefteq \tau}{\mathcal{D}_1 \cup \mathcal{D}_2; \Gamma; \Sigma \vdash e_1 e_2 : \tau}$	T-Set $\frac{\mathcal{D}_1; \Gamma; \Sigma \vdash l \trianglelefteq \Psi \rho \quad \mathcal{D}_2; \Gamma; \Sigma \vdash e \trianglelefteq \rho}{\mathcal{D}_1 \cup \mathcal{D}_2; \Gamma; \Sigma \vdash l := e : \text{unit}}$	
T-If $\frac{\mathcal{D}_1; \Gamma; \Sigma \vdash e_1 \trianglelefteq \text{bool} \quad \mathcal{D}_2; \Gamma; \Sigma \vdash e_2 \trianglelefteq \tau \quad \mathcal{D}_3; \Gamma; \Sigma \vdash e_3 \trianglelefteq \tau \quad \tau' \trianglelefteq \tau}{\mathcal{D}_1 \cup \mathcal{D}_2 \cup \mathcal{D}_3; \Gamma; \Sigma \vdash \text{if } e_1 \text{ then } e_2 \text{ else } e_3 : \tau'}$	T-Deref $\frac{\mathcal{D}; \Gamma; \Sigma \vdash e \trianglelefteq \uparrow \tau}{\mathcal{D}; \Gamma; \Sigma \vdash e^\wedge : \tau}$	
T-Pair $\frac{\mathcal{D}_1; \Gamma; \Sigma \vdash e_1 \trianglelefteq \tau_1 \quad \mathcal{D}_2; \Gamma; \Sigma \vdash e_2 \trianglelefteq \tau_2 \quad \tau'_1 \trianglelefteq \tau_1 \quad \tau'_2 \trianglelefteq \tau_2}{\mathcal{D}_1 \cup \mathcal{D}_2; \Gamma; \Sigma \vdash (e_1, e_2) : \tau'_1 \times \tau'_2}$	T-Sel $\frac{\mathcal{D}; \Gamma; \Sigma \vdash e : \tau \quad \mathbb{N}(\tau) \overset{\nabla}{=} \tau_1 \times \tau_2}{\mathcal{D}; \Gamma; \Sigma \vdash e.i : \tau_i}$	
T-Dup $\frac{\mathcal{D}; \Gamma; \Sigma \vdash e \trianglelefteq \tau \quad \tau' \trianglelefteq \tau \quad \mathcal{P}(e, \tau')}{\mathcal{D}; \Gamma; \Sigma \vdash \text{dup}(e) : \uparrow \tau'}$	T-Let-M $\frac{\mathcal{D}_1; \Gamma; \Sigma \vdash e_1 \trianglelefteq \tau_1 \quad \tau \trianglelefteq \tau_1 \quad \mathcal{P}(x, \tau_1) \quad \mathcal{D}_2; \Gamma, x \mapsto \tau; \Sigma \vdash e_2 : \tau_2}{\mathcal{D}_1 \cup \mathcal{D}_2; \Gamma; \Sigma \vdash (\text{let}^\Psi \underline{x} = e_1 \text{ in } e_2) : \tau_2}$	
T-Let-MP $\frac{\mathcal{D}_1; \Gamma; \Sigma \vdash v \trianglelefteq \tau_1 \quad \tau \trianglelefteq \tau_1 \quad \mathcal{P}(x, \tau_1) \quad \mathcal{D} = \mathcal{D}_1 \cup \{\star_x^\times(\tau)\} \quad \{\bar{\alpha}\} = \text{ftv}(\tau, \mathcal{D}) \setminus \text{ftv}(\Gamma, \Sigma)}{\mathcal{D}_2; \Gamma, x \mapsto \nabla \bar{\alpha}. \tau \setminus \mathcal{D}; \Sigma \vdash e : \tau_2 \quad \frac{}{\text{new}} \bar{\beta}}{\mathcal{D}[\bar{\beta}/\bar{\alpha}] \cup \mathcal{D}_2; \Gamma; \Sigma \vdash (\text{let}^\times \underline{x} = v \text{ in } e) : \tau_2}$		

Figure 4. Declarative Type Rules

I-Unit $\frac{}{\Gamma; \Sigma \vdash () : \text{unit} \mid \emptyset}$	I-Bool $\frac{}{\Gamma; \Sigma \vdash b : \text{bool} \mid \emptyset}$	I-Id $\frac{\Gamma(x) = \forall \bar{\alpha}. \tau \setminus \mathcal{D} \quad \theta = \overline{[\alpha \mapsto \beta]} \quad \frac{}{\models_{\text{new}} \bar{\beta}}}{\Gamma; \Sigma \vdash x : \theta \langle \tau \rangle \mid \theta \langle \mathcal{D} \rangle}$
I-Hloc $\frac{\Sigma(\ell) = \tau}{\Gamma; \Sigma \vdash \ell : \uparrow \tau \mid \emptyset}$	I-Sloc $\frac{\Sigma(l) = \tau}{\Gamma; \Sigma \vdash l : \tau \mid \emptyset}$	I-Lambda $\frac{\Gamma, x \mapsto \mathbb{P}(x, \beta \downarrow \alpha); \Sigma \vdash e : \tau \mid \mathcal{C} \quad \frac{}{\models_{\text{new}} \alpha \beta \beta' \gamma \gamma' \delta}}{\Gamma; \Sigma \vdash \lambda \underline{x}. e : \beta' \downarrow \alpha \rightarrow \gamma' \downarrow \delta \mid \mathcal{C} \cup \{\tau = \gamma \downarrow \delta\}}$
I-App $\frac{\Gamma; \Sigma \vdash e_1 : \tau_1 \mid \mathcal{C}_1 \quad \Gamma; \Sigma \vdash e_2 : \tau_2 \mid \mathcal{C}_2 \quad \frac{}{\models_{\text{new}} \alpha \beta \beta' \gamma \gamma' \delta \varepsilon}}{\Gamma; \Sigma \vdash e_1 e_2 : \varepsilon \downarrow \gamma \mid \mathcal{C}_1 \cup \mathcal{C}_2 \cup \{\tau_1 = \alpha \downarrow (\beta' \downarrow \beta \rightarrow \gamma' \downarrow \gamma), \tau_2 = \delta \downarrow \beta\}}$	I-Deref $\frac{\Gamma; \Sigma \vdash e : \tau \mid \mathcal{C} \quad \frac{}{\models_{\text{new}} \alpha \beta}}{\Gamma; \Sigma \vdash e^\wedge : \alpha \mid \mathcal{C} \cup \{\tau = \beta \downarrow \uparrow \alpha\}}$	
I-If $\frac{\Gamma; \Sigma \vdash e_1 : \tau_1 \mid \mathcal{C}_1 \quad \Gamma; \Sigma \vdash e_2 : \tau_2 \mid \mathcal{C}_2 \quad \Gamma; \Sigma \vdash e_3 : \tau_3 \mid \mathcal{C}_3 \quad \frac{}{\models_{\text{new}} \alpha \beta \gamma \delta \varepsilon}}{\Gamma; \Sigma \vdash \text{if } e_1 \text{ then } e_2 \text{ else } e_3 : \varepsilon \downarrow \gamma \mid \mathcal{C}_1 \cup \mathcal{C}_2 \cup \mathcal{C}_3 \cup \{\tau_1 = \alpha \downarrow \text{bool}, \tau_2 = \beta \downarrow \gamma, \tau_3 = \delta \downarrow \gamma\}}$		
I-Set $\frac{\Gamma; \Sigma \vdash l : \tau_1 \mid \mathcal{C}_1 \quad \Gamma; \Sigma \vdash e : \tau_2 \mid \mathcal{C}_2 \quad \frac{}{\models_{\text{new}} \alpha \beta \gamma}}{\Gamma; \Sigma \vdash l := e : \text{unit} \mid \mathcal{C}_1 \cup \mathcal{C}_2 \cup \{\tau_1 = (\Psi \alpha) \downarrow \beta, \tau_2 = \gamma \downarrow \beta\}}$	I-Dup $\frac{\Gamma; \Sigma \vdash e : \tau \mid \mathcal{C} \quad \tau' = \mathbb{P}(e, \alpha \downarrow \beta) \quad \frac{}{\models_{\text{new}} \alpha \beta \gamma}}{\Gamma; \Sigma \vdash \text{dup}(e) : \uparrow \tau' \mid \mathcal{C} \cup \{\tau = \gamma \downarrow \beta\}}$	
I-Pair $\frac{\Gamma; \Sigma \vdash e_1 : \tau_1 \mid \mathcal{C}_1 \quad \Gamma; \Sigma \vdash e_2 : \tau_2 \mid \mathcal{C}_2 \quad \frac{}{\models_{\text{new}} \alpha \alpha' \beta \beta' \gamma \delta}}{\Gamma; \Sigma \vdash (e_1, e_2) : \alpha \downarrow \gamma \times \beta \downarrow \delta \mid \mathcal{C}_1 \cup \mathcal{C}_2 \cup \{\tau_1 = \alpha' \downarrow \gamma, \tau_2 = \beta' \downarrow \delta\}}$	I-Sel $\frac{\Gamma; \Sigma \vdash e : \tau \mid \mathcal{C} \quad \tau_1 = \alpha \downarrow \beta \quad \tau_2 = \gamma \downarrow \delta \quad \frac{}{\models_{\text{new}} \alpha \beta \gamma \delta \varepsilon}}{\Gamma; \Sigma \vdash e.i : \tau_i \mid \mathcal{C} \cup \{\tau = \varepsilon \downarrow (\tau_1 \times \tau_2)\}}$	
I-Let-Exp $\frac{\Gamma; \Sigma \vdash e_1 : \tau_1 \mid \mathcal{C}_1 \quad e_1 \neq v \quad \Gamma, x \mapsto \mathbb{P}(x, \alpha \downarrow \beta); \Sigma \vdash e_2 : \tau_2 \mid \mathcal{C}_2 \quad \frac{}{\models_{\text{new}} \alpha \beta \gamma \kappa}}{\Gamma; \Sigma \vdash \text{let}^\kappa \underline{x} = e_1 \text{ in } e_2 : \tau_2 \mid \mathcal{C}_1 \cup \{\tau_1 = \gamma \downarrow \beta, \kappa = \psi\} \cup \mathcal{C}_2}$		
I-Let-Val $\frac{\Gamma; \Sigma \vdash v : \tau_1 \mid \mathcal{C}_1 \quad \mathcal{C}'_1 = \mathcal{C}_1 \cup \{\tau_1 = \gamma \downarrow \beta\} \quad \mathcal{U}(\mathcal{C}'_1) = (\mathcal{D}_u, \theta) \quad \mathcal{D} = \mathcal{D}_u \cup \{\star_x^\kappa(\tau)\} \quad \tau = \mathbb{P}(\underline{x}, \theta \langle \delta \downarrow \beta \rangle) \quad \{\bar{\alpha}\} = \text{ftv}(\tau, \mathcal{D}) \setminus \text{ftv}(\theta \langle \Gamma \rangle, \theta \langle \Sigma \rangle) \quad \Gamma, x \mapsto \forall \bar{\alpha}. \tau \setminus \mathcal{D}; \Sigma \vdash e : \tau_2 \mid \mathcal{C}_2 \quad \frac{}{\models_{\text{new}} \beta \gamma \delta \bar{\varepsilon} \kappa}}{\Gamma; \Sigma \vdash \text{let}^\kappa \underline{x} = v \text{ in } e : \tau_2 \mid \mathcal{C}'_1[\bar{\varepsilon}/\bar{\alpha}] \cup \mathcal{C}_2}$		

Figure 5. Type Inference Algorithm

U-Empty	$\mathcal{U}(\emptyset)$	=	$(\emptyset, \langle \rangle)$
U-Refl	$\mathcal{U}(\{\tau = \tau\} \cup \mathcal{C})$	=	$\mathcal{U}(\mathcal{C})$
U-Sym	$\mathcal{U}(\{\tau_1 = \tau_2\} \cup \mathcal{C})$	=	$\mathcal{U}(\{\tau_2 = \tau_1\} \cup \mathcal{C})$
U-Var	$\mathcal{U}(\{\alpha = \tau\} \cup \mathcal{C}) \mid \alpha \notin \tau$	=	$(\mathcal{D}, \theta_a \circ \theta_u)$ where $\theta_a = [\alpha \mapsto \tau]$ and $\mathcal{U}(\mathbb{N}(\theta_a \langle \mathcal{C} \rangle)) = (\mathcal{D}, \theta_u)$
U-Fn	$\mathcal{U}(\{\tau_a \rightarrow \tau_r = \tau'_a \rightarrow \tau'_r\} \cup \mathcal{C})$	=	$\mathcal{U}(\mathcal{C} \cup \{\tau_a = \tau'_a, \tau_r = \tau'_r\})$
U-Ref	$\mathcal{U}(\{\uparrow\tau_1 = \uparrow\tau_2\} \cup \mathcal{C})$	=	$\mathcal{U}(\mathcal{C} \cup \{\tau_1 = \tau_2\})$
U-Mut	$\mathcal{U}(\{\Psi\rho_1 = \Psi\rho_2\} \cup \mathcal{C})$	=	$(\mathcal{D}, \theta_u \circ \theta_m)$ where $\mathbb{M}(\theta_u \langle \tau_1 \rangle) = \theta_m$ and $\mathcal{U}(\mathcal{C} \cup \{\rho_1 = \rho_2\}) = (\mathcal{D}, \theta_u)$
U-Pair	$\mathcal{U}(\{\tau_1 \times \tau_2 = \tau'_1 \times \tau'_2\} \cup \mathcal{C})$	=	$\mathcal{U}(\mathcal{C} \cup \{\tau_1 = \tau'_1, \tau_2 = \tau'_2\})$
U-Const1	$\mathcal{U}(\{\lfloor \tau_1 \rfloor = \lfloor \tau_2 \rfloor\} \cup \mathcal{C})$	=	$\mathcal{U}(\mathcal{C} \cup \{\tau_1 \overset{\circ}{=} \tau_2\})$
U-Const2	$\mathcal{U}(\{\lceil \tau_1 \rceil = \tau_2\} \cup \mathcal{C})$ where $\mathbb{N}(\tau_1) \neq \tau_1$	=	$\mathcal{U}(\mathcal{C} \cup \{\mathbb{N}(\tau_1) = \tau_2\})$
U-Ct1	$\mathcal{U}(\{\alpha \downarrow \rho_1 = \beta \downarrow \rho_2\} \cup \mathcal{C})$	=	$\mathcal{U}(\mathcal{C} \cup \{\rho_1 \overset{\nabla}{=} \rho_2, \alpha = \beta\})$
U-Ct2	$\mathcal{U}(\{\alpha \downarrow \rho = \rho'\} \cup \mathcal{C})$	=	$\mathcal{U}(\mathcal{C} \cup \{\rho \overset{\nabla}{=} \rho', \alpha = \rho'\})$
U-Ct3	$\mathcal{U}(\{\alpha \downarrow \rho_1 = \varsigma \downarrow \rho_2\} \cup \mathcal{C})$	=	$\mathcal{U}(\mathcal{C} \cup \{\rho_1 \overset{\circ}{=} \rho_2, \alpha = \varsigma\})$
U-Ct4	$\mathcal{U}(\{\Psi\alpha \downarrow \rho_1 = \Psi\beta \downarrow \rho_2\} \cup \mathcal{C})$	=	$\mathcal{U}(\mathcal{C} \cup \{\rho_1 \overset{\circ}{=} \rho_2, \alpha = \beta\})$
U-Ct5	$\mathcal{U}(\{\varsigma \downarrow \rho = \varrho\} \cup \mathcal{C})$	=	$\mathcal{U}(\mathcal{C} \cup \{\rho \overset{\circ}{=} \varrho, \varsigma = \varrho\})$
U-K	$\mathcal{U}(\{\kappa = \varkappa\} \cup \mathcal{C})$	=	$(\mathcal{D}, \theta_k \circ \theta_u)$ where $\theta_k = [\kappa \mapsto \varkappa]$ and $\mathcal{U}(\theta_k \langle \mathcal{C} \rangle) = (\mathcal{D}, \theta_u)$
U-Om1	$\mathcal{U}(\{\star_x^\psi(\tau_1), \star_x^\psi(\tau_2)\} \cup \mathcal{C})$	=	$(\mathcal{D} \cup \theta\{\star_x^\psi(\tau_1), \star_x^\psi(\tau_2)\}, \theta)$ where $\mathcal{U}(\mathcal{C} \cup \{\tau_1 = \tau_2\}) = (\mathcal{D}, \theta)$
U-Op1	$\mathcal{U}(\{\star_x^\forall(\tau)\} \cup \mathcal{C}) \mid \Box(\tau)$	=	$(\mathcal{D} \cup \theta\{\star_x^\forall(\tau)\}, \theta)$ where $\mathcal{U}(\mathcal{C} \cup \{\tau = \mathcal{I}(\tau)\}) = (\mathcal{D}, \theta)$
U-Om2	$\mathcal{U}(\{\star_x^\kappa(\tau)\} \cup \mathcal{C}) \mid \text{Mut}(\tau)$	=	$(\mathcal{D}, \theta_k \circ \theta_u)$ where $\theta_k = [\kappa \mapsto \psi]$ and $\mathcal{U}(\theta_k \langle \{\star_x^\kappa(\tau)\} \cup \mathcal{C} \rangle) = (\mathcal{D}, \theta_u)$
U-Op2	$\mathcal{U}(\{\star_x^\kappa(\tau_1), \star_x^\kappa(\tau_2)\} \cup \mathcal{C})$ where $\mathcal{U}(\{\tau_1 = \tau_2\} \cup \mathcal{C}) = \perp$	=	$(\mathcal{D}, \theta_k \circ \theta_u)$ where $\theta_k = [\kappa \mapsto \forall]$ and $\mathcal{U}(\theta_k \langle \{\star_x^\kappa(\tau_1), \star_x^\kappa(\tau_2)\} \cup \mathcal{C} \rangle) = (\mathcal{D}, \theta_u)$
U-Error	$\mathcal{U}(c \cup \mathcal{C}) \mid c \notin \mathcal{C}_v \cup \mathcal{C}_s \cup \mathcal{C}_p$	=	\perp
	$\mathcal{C}_v = \forall \alpha, \varsigma, \rho, \tau, \tau' \mid \alpha \notin \tau'. \{\tau = \tau, \alpha = \tau', \tau' = \alpha, \alpha \downarrow \rho = \tau, \tau = \alpha \downarrow \rho, \varsigma \downarrow \rho = \tau, \tau = \varsigma \downarrow \rho\}$		
	$\mathcal{C}_s = \forall \rho, \rho', \tau, \tau', \tau_1, \tau'_1. \{\tau \rightarrow \tau_1 = \tau' \rightarrow \tau'_1, \uparrow\tau = \uparrow\tau', \Psi\rho = \Psi\rho', \lfloor \tau \rfloor = \lfloor \tau' \rfloor\}$		
	$\mathcal{C}_p = \forall x, \kappa, \varkappa, \tau, \tau' \mid \neg \text{Mut}(\tau'). \{\kappa = \varkappa, \star_x^\psi(\tau), \star_x^\forall(\tau'), \star_x^\kappa(\tau)\}$		

Figure 6. Unification Algorithm